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MA222 - Computational Linear Algebra  
Problem Sheet - 1

Basic Algorithms and Notation

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1. Suppose  $A \in \mathbb{R}^{n \times n}$  and  $x \in \mathbb{R}^r$  are given. Give a saxpy algorithm for computing the first column of  $M = (A - x_1 I) \cdots (A - x_r I)$ .
2. In the conventional 2-by-2 matrix multiplication  $C = AB$ , there are eight multiplications:  $a_{11}a_{11}, a_{11}b_{12}, a_{21}b_{11}, a_{21}b_{12}, a_{12}b_{21}, a_{12}b_{22}, a_{22}b_{21}$ , and  $a_{22}b_{22}$ . Make a table that indicates the order that these multiplications are performed for the  $ijk, jik, kij, ikj, jki$ , and  $kji$  matrix multiply algorithms.
3. Give an algorithm for computing  $C = (xy^T)^k$  where  $x$  and  $y$  are  $n$ -vectors.
4. Specify an algorithm for computing  $(XY^T)^k$  where  $X, Y \in \mathbb{R}^{n \times 2}$ .
5. Formulate an outer product algorithm for the update  $C = AB^T + C$  where  $A \in \mathbb{R}^{m \times r}, b \in \mathbb{R}^{n \times r}$ , and  $C \in \mathbb{R}^{m \times n}$ .
6. Suppose we have real  $n$ -by- $n$  matrices  $C, D, E$ , and  $F$ . Show how to compute real  $n$ -by- $n$  matrices  $A$  and  $B$  with just three real  $n$ -by- $n$  matrix multiplications so that  $(A + iB) = (C + iD)(E + iF)$ . Hint: Compute  $W = (C + D)(E - F)$ .

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